

## 2 and 3-point functions in the ENJL-model<sup>1</sup>.

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### Abstract

We discuss the extended Nambu-Jona-Lasinio model as a low energy expansion, all two-point functions and an example of a three-point function to all orders in momenta and quark masses. The model is treated at leading level in  $1/N_c$  but otherwise exact. Some comments about the QCD flavour anomaly and Vector Meson Dominance in this class of models is made.

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# 1 Introduction

The extended Nambu-Jona-Lasinio (ENJL) model has phenomenologically been very successful. For a recent review see [1]. It is a typical example of a class of low energy hadronic models with quarks that incorporates chiral symmetry correctly. The aim of this investigation was twofold:

1) to understand the relation between chiral symmetry and the concept of constituent quarks.

2) to understand why this class of models works so well phenomenologically.

The main conclusions are:

1) in a sense the constituent quark mass is the same as the quark-anti-quark vacuum expectation value.

2) The phenomenological success rests on 3 bases. The use of the  $1/N_c$  expansion naturally leads to a kind of generalized meson dominance. The short-distance behaviour of these models has a lot in common with QCD and a large number of relations are fairly independent of the method of regularization used.

The ENJL model has been treated for a long time and by many authors, see the list of references in [1, 2]. The Lagrangian is given by

$$\begin{aligned} \bar{q} [i\gamma^\mu (\partial_\mu - iv_\mu - ia_\mu \gamma_5) - \mathcal{M} - s + ip\gamma_5] q \\ + 2g_S \sum_{i,j} (\bar{q}_R^i q_L^j) (\bar{q}_L^j q_R^i) \\ - g_V \sum_{i,j} |\bar{q}_L^i \gamma_\mu q_L^j|^2 + |\bar{q}_R^i \gamma_\mu q_R^j|^2, \end{aligned} \quad (1)$$

with  $\bar{q} = (\bar{u} \ \bar{d} \ \bar{s})$  and  $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$ .  $g_S$  and  $g_V$  are given by  $g_{S(V)} = 4(8)\pi^2 G_{S(V)}/(N_c \Lambda_\chi^2)$  in terms of the notation used in [2, 3, 4, 5].  $v_\mu, a_\mu, s$  and  $p$  are external vector, axial-vector, scalar and pseudoscalar fields. These are used to probe the theory with. This Lagrangian can be argued to follow from QCD in the following way: all indications are that in the pure glue sector there is a mass gap. The lowest glueball has a mass of about 1.5 GeV. This means that correlations below this scale should vanish. We then treat the interactions of quarks below this scale as pointlike. It can also be seen as the first terms in an expansion in local terms after integrating out fully (or only the short distance part) of the gluons. These are in fact all terms up to dimension 6 to leading order in  $1/N_c$ . The Lagrangian in (1) also has the correct chiral symmetry transformations.

Our first conclusion can be seen by looking at the Schwinger-Dyson equation for the propagator depicted in Fig. 1. This leads to the equation for the constituent

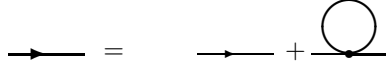


Figure 1: The Schwinger Dyson equation for the propagator. A thin (thick) line is the bare (full) fermion propagator.

quark mass ( $M_i$ ) in terms of the current one ( $m_i$ ):

$$\begin{aligned} M_i &= m_i - g_S \langle \bar{q} q \rangle_i \\ \langle \bar{q} q \rangle_i &= -N_c 4 M_i \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2 - M_i^2}. \end{aligned} \quad (2)$$

The solution is shown in Fig. 2. For nonzero current quark mass there is only one solution. This one corresponds for  $m_i \rightarrow 0$  to the one with a nonzero value for  $\langle \bar{q} q \rangle$  so that the chiral symmetry is spontaneously broken. It also approaches this case smoothly so chiral perturbation theory should be a valid expansion in this model.

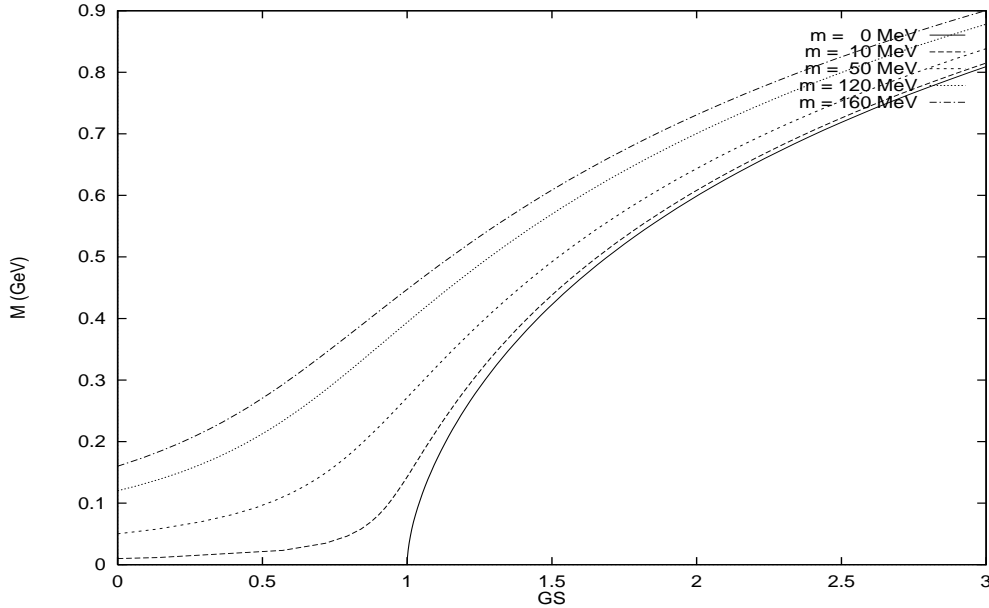


Figure 2: Plot of the dependence of the constituent quark mass  $M_i$  as a function of  $G_S$  for several values of  $m_i$

## 2 Low-energy expansion

We use here the formulas of [2]. We show here the results for the low-energy parameters of chiral perturbation theory. Fitted are the  $\mathcal{O}(p^4)$  parameters  $L_i$  that are the free parameters at next-to-leading order. They are defined in [2]. The scale is set by the only dimensionful parameter we fit here, the pion decay constant in the chiral limit,  $f_0 \approx 86$  MeV. We have redone the fits for a few new cases. One where we enforce the one-gluon exchange relation  $G_S = 4G_V$ , The one in the renormalon picture (Ref. [6]) or  $G_V = 0$  and with both  $G_V$  and  $G_S$  free. These are column 3, 4 or 5 respectively in Table 1. Notice that the fit is significantly worse with  $G_V = 0$ . In [2] this fit was better due to the presence of a gluonic vacuum expectation value. The value of the fitted cut-off also decreases significantly with lower  $G_V$  values. The value of  $G_S$  is determined from  $M_Q$  and  $\Lambda_\chi$  via (2).

In addition to the good numerical fit a set of relations were obtained that were

	exp			
$G_V$	-	$G_S/4$	0	1.264
$M_Q(\text{MeV})$	-	260	280	265
$\Lambda_\chi(\text{MeV})$	-	810	630	1160
$10^3 L_2$	1.2	1.5	1.6	1.6
$10^3 L_3$	-3.6	-3.1	-3.0	-4.1
$10^3 L_5$	1.4	2.1	1.9	1.5
$10^3 L_8$	0.9	0.9	0.8	0.8
$10^3 L_9$	6.9	5.7	5.2	6.7
$10^3 L_{10}$	-5.5	-3.9	-2.6	-5.5
$M_V(\text{MeV})$	770	1260	$\infty$	810
$M_A(\text{MeV})$	$\approx 1260$	2010	$\infty$	1330

Table 1: Best fit values for the low energy parameters using several constraints on  $G_V$ .

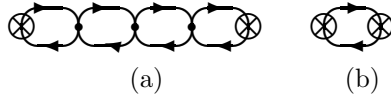


Figure 3: The graphs contributing to the two point-functions in the large  $N_c$  limit. a) The class of all strings of constituent quark loops. The four-fermion vertices are those of 1. The crosses at both ends are the insertion of the external sources. b) The one-loop case.

independent of the regularization. Some of these are:

$$L_9 = \frac{1}{2} f_V g_V \quad (3)$$

$$f_\pi^2 = f_V^2 m_V^2 - f_A^2 m_A^2 . \quad (4)$$

The former is the same as the vector meson dominance expression for the pion vector form factor and the latter is the first Weinberg sum rule. The question arises how general are these relations and are they broken at higher orders ? Therefore we went on to analyze two-point functions to all orders in masses and momenta.

### 3 Two-point functions to all orders in $q^2$ and $m_q$ .

The chiral limit case was analyzed in [3], the corrections due to nonzero quark masses can be found in [5]. Several relations were found to be true to all orders. As an example we will derive here the relation between the scalar mass and the constituent quark mass in the chiral limit. The set of diagrams that contributes is drawn in Fig. 3a. The series can be rewritten as a geometric series and can be easily summed in terms of the one-loop 2-point function  $\bar{\Pi}_S$ . The full result for the scalar-scalar two-point function (we only treat the case with equal masses here, see [5] for the general case) is:

$$\Pi_S = \frac{\bar{\Pi}_S}{1 - g_S \bar{\Pi}_S} . \quad (5)$$

The resummation has generated a pole that corresponds to a scalar particle. Can we say more already at this level? We can in fact. The Ward identities for the one loop functions are:

$$\bar{\Pi}_S = \bar{\Pi}_P - q^2 \bar{\Pi}_A^{(0)} , \quad (6)$$

$$\bar{\Pi}_P = \frac{q^4}{4M_Q^2} \bar{\Pi}_A^{(0)} - \frac{\langle \bar{q}q \rangle}{M_Q} . \quad (7)$$

(6) is a consequence of using the heat kernel for the one-loop functions and (7) is a direct consequence of the symmetry. Using these two relations we can rewrite

$$1 - g_S \bar{\Pi}_S = 1 + \frac{g_S}{M_Q} + (q^2 - 4M_Q^2) \frac{q^2 \bar{\Pi}_A^{(0)}}{4M_Q^2} . \quad (8)$$

The first two terms vanish due to the gap equation so this two-point function has a pole at twice the constituent mass. For nonvanishing current quark masses there is a small correction

$$M_S^2 = 4M_Q^2 + g_A(-M_S^2)m_{ii}(-M_S^2) . \quad (9)$$

See [5] for definitions. Other examples of relations with the same range of validity are:

1. The first and second Weinberg sum rule are satisfied, indicating a somewhat too suppressed high energy behaviour for the last one.
2. The third Weinberg sum rule is violated as in QCD.
3. The Gell-Mann-Oakes-Renner relation to all orders in  $m_q$  reads  $2m_\pi^2(-q^2)f_\pi^2(-q^2) = (m_i + m_j)(M_i + M_j)/g_S$ .

These are valid in all schemes where the one-loop functions are obtained from a heat kernel like expansion and have thus a rather broad range of validity. In particular, they remain valid at finite temperatures and densities.

## 4 3 point functions and anomalies

We discuss in this section as an example the pseudoscalar-vector-vector 3-point function. This has all the interesting features plus the occurrence of the flavour anomaly. The general diagram is a one-loop triangle diagram with a chain with 0,1,2,3,... one-loop (like in Fig. 3a) connected to all three corners. The two vector legs can be easily resummed leading to an expression like:

$$\frac{g_{\mu\alpha}M_V^2(-p_1^2) - p_{1\mu}p_{1\alpha}}{M_V^2(p_1^2) - p_1^2} \quad (10)$$

The resummation of the external leg leads immediately to a VMD-like formula in terms of slowly varying functions of the momenta. A similar expression is valid for the other vector leg. The pseudoscalar leg can mix with the longitudinal axial-vector degree of freedom leading to a sum of two terms. Both with a pole at the pseudoscalar mass. Naive use of the current identity  $-iq^\alpha \bar{q}\gamma_\alpha\gamma_5 q = 2M_Q i\bar{q}\gamma_5 q$  would lead to only the pseudoscalar term multiplied by  $g_A(-q^2)$ . It is in this way that this resummation method sees the mixing of the pion and the axial-vector that occurs in this method.

In fact two more effects should be taken into account. The current identity is used in a three-point function so there are usually also terms from the equal-time commutators and there are additional terms in the current identity due to the anomaly. These latter are very important in obtaining results that have the correct QCD flavour anomaly [4, 5]. The final result for the PVV two-point function with momenta  $p_{1,2}$  for the vector legs is:

$$\begin{aligned} \Pi_{\mu\nu}^{PVV}(p_1, p_2) = & \frac{N_c}{16\pi^2} \frac{\varepsilon_{\mu\nu\beta\rho} p_1^\beta p_2^\rho 4M_Q}{g_S f_\pi^2(q^2)(m_\pi^2(q^2) - q^2)} \\ & \left\{ \frac{M_V^2(p_1^2)M_V^2(p_2^2)g_A(q^2)}{(M_V^2(p_1^2) - p_1^2)(M_V^2(p_2^2) - p_2^2)} F(q^2, p_1^2, p_2^2) \right. \\ & \left. + 1 - g_A(q^2) \right\} \quad (11) \end{aligned}$$

The function  $F(q^2, p_1^2, p_2^2)$  is essentially the chiral quark loop result. So analytically we only have a part that is multiplied by the expected Vector-Meson-Dominance factors. There is a second part that is not, that came from the extra terms in the current identity. This behaviour is in fact very welcome. We have both the one-loop quark contribution to the slopes and the one from vector meson

dominance. Since both of these explain the observed slopes having both fully would not agree with experiment. Here we have, however  $F_{PVV}(m_\pi^1, p_1^2, p_2^2) \approx 1 + \rho(p_1^2 + p_2^2) + \rho' m_\pi^2 + \dots$  with  $\rho = g_A(0) (1/M_V^2(0) + 1/(12M_Q^2)) \approx 1.53 \text{ GeV}^{-2}$  and  $\rho' = (g_A(0)/(12M_Q^2)) (1 - \Gamma(1)/\Gamma(0)) \approx 0.40 \text{ GeV}^{-2}$ . As we see we have good numerical agreement with the observed slope and the corrections due to finite meson mass are substantially smaller than in the chiral quark model. The latter is also desirable since otherwise there would have been extremely large corrections to the  $\eta$  decay.

## 5 Meson Dominance

As shown in the previous two sections the appearance of meson dominance like formulas with slowly varying couplings is a natural feature of this model and as such the successful phenomenology of this concept is taken over. The model does combine this together with a set of chiral quark loop effects in a kind of interpolating fashion thus incorporating the strengths of both approaches. The final results can be plotted to check whether the final formulas also have a VMD-like behaviour and as shown in section 5 in [5] this is numerically the case for all the 2 and 3-point functions studied there. We have in general stayed in the euclidean domain of momenta to avoid the problem that this model does not include confinement. There the sign of meson dominance is that inverse formfactors are straight lines as a function of the various  $q^2$ . This we find indeed.

## 6 Conclusions

We have treated the ENJL model in leading order in  $1/N_c$  as an a low-energy expansion for a general Green function via chiral perturbation theory and all 2-point functions and a few 3-point functions to all orders in quark masses and momenta. We have also given a consistent treatment of the QCD flavour anomaly in this model. So far the comparison with wanted QCD results and experiment has been very successful.

## References

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